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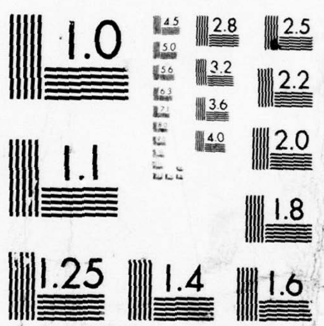

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THE LEAST-SQUARES ESTIMATION OF LATENT TRAIT VARIABLES BY A HILBERT SPACE APPROACH

KIKUMI TATSUOKA

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This research developed a new method for estimating a given latent trait variable θ by the least-squares approach. The notion of multiple regression equation was reinterpreted in terms of properties of a Hilbert space and the calculation formula for β weights that can be obtained recursively in the form of Fourier series was derived. The θ values estimated by this method and the maximum likelihood method were compared using live data. → next page

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Theta

It was shown that θ values estimated by the least-squares method was just as good as θ by the maximum likelihood method. The advantage of using this method as against the traditional method is that values of θ are always obtainable even for a small number of items. The maximum likelihood method, on the other hand, often fails to converge in such cases.

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by Kikumi Tatsuoka

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ABSTRACT

This research developed a new method for estimating a given latent trait variable θ by the least-squares approach. The notion of multiple regression equation was reinterpreted in terms of the properties of a Hilbert space and the calculation formula for β weights that can be obtained recursively in the form of Fourier series was derived. The θ values estimated by this method and the maximum likelihood method were compared using live data. It was shown that $\hat{\theta}$ values estimated by the least-squares method was just as good as $\hat{\theta}$ by the maximum likelihood method. The advantages of using this method as against the traditional method is that values of $\hat{\theta}$ are always obtainable even for a small number of items. The maximum likelihood method, on the other nand, often fails to converge in such cases.

INTRODUCTION

Latent trait theory has provided the only models that are applicable to assessing an individual student's ability level in an adaptive testing situation where each student takes a different set of items. The estimation of parameters of the latent trait theory models has been achieved iteratively by the maximum likelihood (MXL) method. This method provides considerably accurate values for data generated by the Monte Carlo method, but it often fails to converge and cannot estimate the parameters for live data in which even only a very few instances of certain kinds of erroneous data were unfortunately included; it also fails for either very high or very low θ values. This sensitivity leads us to an inconvenient position when adaptive testing will be used in practice, either as a diagnostic test or a posttest at the end of instruction. When diagnostic adaptive testing is used for routing each student to his/her most appropriate instructional unit in a series of lessons written on computer-based education systems, the problem of having nonconvergent θ values causes disasters.

Several diagnostic adaptive tests (e.g., operations with signed numbers and matrix algebra lessons) were implemented on the PLATO system and they have been administered at a junior high school and the University of Illinois this fall semester. The results from these tests were used for routing students to their levels in the instructional materials prepared on the system. The maximum likelihood method failed to provide estimated θ values for about 10 percent of the students. A part of the blame might be attributed to the fact that the U of I system of Computer Based Education

(CBE) is more widely used for those who are either very advanced or who are slow learners than for average students at the public school systems in Champaign and Urbana. And also the number of subjects that was used to calibrate item discriminating powers and difficulties was far less than the 2000 which has been customarily required. It is, indeed, impossible to collect as many as 2000 subjects' data with our CBE system, especially in order to develop diagnostic or criterion referenced adaptive testings as measures of achievement either before or after a series of instructions--even though it is well known that the PLATO system at the U of I has the largest usage in the country.

The purpose of this study is to propose a new model that will take advantage of the powerful properties of a Hilbert space, where the estimated θ values will be always obtained. The coefficients, i.e., β weights in the multiple regression equation--which, in the Hilbert space approach, are obtained by projecting θ onto the space spanned by given predictor vectors--are used to estimate θ with the help of Fourier series. The results are compared with those obtained by the MXL estimation method.

The accuracy of the newly presented least-squares (LS) estimation method of θ is discussed in the following sections. It can be shown that the properties of multiple R and standard error of estimate developed in terms of norms and Fourier coefficients exactly parallel those from the theory of multiple regression.

MULTIPLE REGRESSION FUNCTIONS AND PROJECTION OPERATORS

Hilbert space: Suppose $\underset{\sim}{X}_g$ is the raw-score vector of a given item g in a

population, or more precisely a space spanned by the persons in the population. This space can be a complete latent space of the unidimensional latent trait θ for a given set of n items. In latent trait theory, a response pattern for person i is crucial for determining the likelihood function, but the focus of our interest in this paper will be switched to the score vector of item g in the person-space where the two parameter latent trait model is considered. Let us consider a deviation score vector $\underset{\sim}{x}_g = (X_{g1}, X_{g2}, \ldots X_{gN}) - (\mu_g, \mu_g, \ldots, \mu_g)$ for convenience in the further work. A set of deviation score vectors $\{x_i\}$ will be called a Hilbert space if an appropriate metric, or norm is defined.

If the norm of $\underset{\sim}{x}_g$ is defined by

$$\|\underset{\sim}{x}_g\| = \sqrt{\sum_{i=1}^{N} x_{gi} x_{gi}} = (\underset{\sim}{x}_g, \underset{\sim}{x}_g)^{1/2},$$

then the standard deviation of item g becomes to the norm of $\underset{\sim}{x}_g$ divided by $\sqrt{N-1}$, and the covariance of two vectors $\underset{\sim}{x}_g$ and $\underset{\sim}{x}_f$ is the inner product of these two vectors divided by N-1. Two vectors $\underset{\sim}{x}_g$ and $\underset{\sim}{x}_f$ are said to be orthogonal if their inner product equals zero, that is $(\underset{\sim}{x}_g, \underset{\sim}{x}_f) = 0$. A vector $\underset{\sim}{x}_g$ is said to be orthogonal to a set M if $\underset{\sim}{x}_g$ is orthogonal to every vector in M. Hilbert space with these properties which are direct consequences from the definition of the inner product of any two vectors provides a wealth of structural characteristics that implies very useful analytical results applicable to many problems. For example, the shortest vector from a point to a subspace M in n-dimensional Euclidean space is orthogonal to the subspace M. This is a special case of the optimization principle which is called the projection theorem. Tatsuoka (1975) discussed in detail

about the relationship among projection operators, regression functions and multiple regression functions in her paper, so only a brief explanation will be given here.

The projection theorem can be explained briefly as follows: For a given vector $\underset{\sim}{x}$ and a subspace M, there exists a unique vector $\underset{\sim}{m}$ in M which is closest to $\underset{\sim}{x}$ in the sense that it minimizes $\|\underset{\sim}{x} - \underset{\sim}{m}\|$. Furthermore a necessary and sufficient condition that $\underset{\sim}{m}$ be the unique minimizing vector is that $\underset{\sim}{x} - \underset{\sim}{m}$ be orthogonal to M. Hence, $\underset{\sim}{m}$ can be called the multiple regression function $R(\underset{\sim}{x} \mid M)$ and the residual will be $\underset{\sim}{x} - \underset{\sim}{m}$ which is orthogonal to $\underset{\sim}{m} = R(\underset{\sim}{x} \mid M)$. If M is of a single vector space then $R(\underset{\sim}{x} \mid M)$ is the regression function of $\underset{\sim}{x}$ onto M. It is always true that $R(\underset{\sim}{x} \mid M)$ is linear and idempotent; that is tor any real variables s and t,

$$R(s\underset{\sim}{x}_g + t\underset{\sim}{x}_f \mid M) = sR(\underset{\sim}{x}_g \mid M) + tR(\underset{\sim}{x}_f \mid M)$$

$$R(R(\underset{\sim}{x}_g \mid M) \mid M) = R(\underset{\sim}{x}_g \mid M).$$

In order to express $R(\underset{\sim}{x}_g \mid M)$ in the practical form of a linear function, we will need to introduce more properties briefly in Hilbert space. The notion of orthonormal basis, Fourier series will be introduced in the following.

<u>An orthonormal set of vectors</u>: A set of vectors $\underset{\sim}{e}_i$, i= 1, . . .n (denoted by $\{\underset{\sim}{e}_i\}$ from now on) is said to be orthogonal if each vector in the set is orthogonal and has norm equal to unity. An orthonormal set of nonzero vectors are mutually independent; in other words, the inner product of any two vectors in the set equals zero. In Hilbert space, such a set always exists

and can be easily constructed by the Gram-Schmidt orthogonalization procedure. It is clear that the $\underset{\sim}{e}_i$s in the set of orthogonal vectors generate the same space as that generated by the $\underset{\sim}{x}_i$s.

<u>Normal equations in the context with Hilbert space</u>: Our concern in the study is to express the multiple regression of $\underset{\sim}{\theta}$ onto set $\{\underset{\sim}{x}_i\}$ as a linear combination of $\underset{\sim}{x}_i$s, $\hat{\underset{\sim}{\theta}} = d_1\underset{\sim}{x}_1 + d_2\underset{\sim}{x}_2 + \ldots + d_n\underset{\sim}{x}_n$ which is equivalent to that of finding the n scalars d_i, $i = 1, \ldots, n$, upon minimizing expression (1),

(1) $$\underset{\sim}{\theta} - d_1\underset{\sim}{x}_1 - d_2\underset{\sim}{x}_2 - \ldots - d_n\underset{\sim}{x}_n$$

or, setting the inner product equal to zero,

(2) $$(\underset{\sim}{\theta} - d_1\underset{\sim}{x}_1 - d_2\underset{\sim}{x}_2 - \ldots - d_n\underset{\sim}{x}_n, \underset{\sim}{x}_i) = 0$$

for each i. By rewriting Equation (2), we obtain Equation (3).

(3) $$\begin{aligned} d_1(\underset{\sim}{x}_1,\underset{\sim}{x}_1) + d_2(\underset{\sim}{x}_2,\underset{\sim}{x}_1) + \ldots + d_n(\underset{\sim}{x}_n,\underset{\sim}{x}_1) &= (\underset{\sim}{\theta},\underset{\sim}{x}_1) \\ d_1(\underset{\sim}{x}_1,\underset{\sim}{x}_2) + d_2(\underset{\sim}{x}_2,\underset{\sim}{x}_2) + \ldots + d_n(\underset{\sim}{x}_n,\underset{\sim}{x}_2) &= (\underset{\sim}{\theta},\underset{\sim}{x}_2) \\ \ldots\ldots\ldots\ldots\ldots & \\ d_1(\underset{\sim}{x}_1,\underset{\sim}{x}_n) + d_2(\underset{\sim}{x}_2,\underset{\sim}{x}_n) + \ldots + d_n(\underset{\sim}{x}_n,\underset{\sim}{x}_n) &= (\underset{\sim}{\theta},\underset{\sim}{x}_n) \end{aligned}$$

This set of n equations with n coefficients d_is is known as the normal equations for the least-squares problems if $\underset{\sim}{\theta}$ is replaced by any criterion vector. If the n x n matrix of $(\underset{\sim}{x}_i,\underset{\sim}{x}_j)$, $i,j = 1, \ldots n$ has a nonzero determinant, then Equation (3) can be solved for the coefficients d_i. Since

$(\underset{\sim}{\theta},\underset{\sim}{x}_i) = (N-1)\rho_{\theta i}\sigma(\underset{\sim}{\theta})\sigma(\underset{\sim}{x}_i)$, the right-hand sides of Equations (3) will be replaced by the known values $a_i(N-1)\sigma(\underset{\sim}{x}_i)/\sqrt{(1+a_i^2)}$ (Lord and Novick, 1968) if we assume $\underset{\sim}{\theta} \sim N(0,1)$, where a_i is the item discriminating power of item i.

The evaluation of an n x n determinant is not an easy task, especially if n is large. As an alternative approach, we will introduce an analytical method using Fourier series to estimate $\underset{\sim}{\theta}$.

Fourier series and Gram-Schmidt procedure: Let us apply the Gram-Schmidt procedure to a set of $\{\underset{\sim}{x}_1,\underset{\sim}{x}_2,\ldots,\underset{\sim}{x}_n\}$ (or $\{\underset{\sim}{x}_i\}$), obtaining an orthonormal set $\{\underset{\sim}{e}_1,\underset{\sim}{e}_2,\ldots,\underset{\sim}{e}_n\}$. Then any vector $\underset{\sim}{y}$ in the space is approximated in the form of a Fourier series (Simmons, 1963; Dunford and Schwartz, 1964; Tatsuoka, 1975) as follows,

$$(4) \qquad \hat{\underset{\sim}{y}} = \sum_{i=1}^{n} (\underset{\sim}{y},\underset{\sim}{e}_i)\underset{\sim}{e}_i$$

Since $\underset{\sim}{y} - \hat{\underset{\sim}{y}}$ is orthogonal to the space spanned by $\{\underset{\sim}{e}_i\}$, which is equivalent to saying that $\underset{\sim}{y} - \hat{\underset{\sim}{y}}$ is orthogonal to the space spanned by $\{\underset{\sim}{x}_i\}$, the estimated y will be easily obtained once the independent vectors $\underset{\sim}{x}_i$s are orthogonalized. Equation (4) relates closely with the Gram-Schmidt procedure because the numerator of Equation (5) below is the residual vector $\underset{\sim}{x}_k - \hat{\underset{\sim}{x}}_k$, where $\hat{\underset{\sim}{x}}_k$ is the regression of $\underset{\sim}{x}_k$ onto the space M_{k-1} spanned by $\{\underset{\sim}{x}_{k-1}\}$ or equivalently by $\{\underset{\sim}{e}_{k-1}\}$ and moreover $\underset{\sim}{x}_k - \hat{\underset{\sim}{x}}_k$ is orthogonal to space M_{k-1}.

$$(5) \qquad \underset{\sim}{e}_i = \frac{\underset{\sim}{x}_i - R(\underset{\sim}{x}_i \mid \underset{\sim}{e}_1,\underset{\sim}{e}_2,\ldots,\underset{\sim}{e}_{i-1})}{\|\underset{\sim}{x}_i - R(\underset{\sim}{x}_i \mid \underset{\sim}{e}_1,\underset{\sim}{e}_2,\ldots,\underset{\sim}{e}_{i-1})\|}$$

$$= \frac{\underset{\sim}{x}_i - R(\underset{\sim}{x}_i \mid \underset{\sim}{e}_1,\underset{\sim}{e}_2,\ldots,\underset{\sim}{e}_{i-1})}{K_i}, \text{ say.}$$

$\underset{\sim}{e}_i$, i = 1,. . .,n are normalized residual vectors while $\underset{\sim}{x}_k - \hat{\underset{\sim}{x}}_k$ are not. If we consider the space spanned by $\{\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n, \underset{\sim}{\theta}\}$, adding $\underset{\sim}{\theta}$ to $\{\underset{\sim}{x}_k\}$ then the last orthogonalized vector $\underset{\sim}{e}_{n+1}$ by Gram-Schmidt procedure has the numerator of $\underset{\sim}{\theta} - \hat{\underset{\sim}{\theta}}$, where the estimated $\underset{\sim}{\theta}$ is given by Equation (6) below, replacing vector $\underset{\sim}{y}$ in Equation (4) by $\underset{\sim}{\theta}$.

$$\hat{\underset{\sim}{\theta}} = \sum_{i=1}^{n} (\underset{\sim}{\theta}, \underset{\sim}{e}_i)\underset{\sim}{e}_i \tag{6}$$

Also, the Gram-Schmidt process can be said to be a procedure for inverting the matrix given by Equation (3) and can itself be viewed as the solution of a minimum norm approximation or the least-squares problem.

ACCURACY OF THE ESTIMATION

Standard error of prediction: The variance error of prediction is 1/(N-1) times the squared norm of the residual vector $\underset{\sim}{\theta} - \hat{\underset{\sim}{\theta}}$ and also is the difference of the squared norms of $\underset{\sim}{\theta}$ and $\hat{\underset{\sim}{\theta}}$.

$$\begin{aligned} \sum_{i=1}^{N} (\underset{\sim}{\theta}_i - \hat{\underset{\sim}{\theta}}_i)^2 &= (\underset{\sim}{\theta} - \hat{\underset{\sim}{\theta}}, \underset{\sim}{\theta} - \hat{\underset{\sim}{\theta}}) \\ &= (\underset{\sim}{\theta}, \underset{\sim}{\theta}) - (\hat{\underset{\sim}{\theta}}, \hat{\underset{\sim}{\theta}}) \\ &= \|\underset{\sim}{\theta}\|^2 - \|\hat{\underset{\sim}{\theta}}\|^2 \end{aligned} \tag{7}$$

where N is the number of subjects. The standard error of prediction is the square root of this.

Equation (7) describes a relation between the estimated $\underset{\sim}{\theta}$ and $\hat{\underset{\sim}{\theta}}$. If the norm of both $\underset{\sim}{\theta}$ and $\hat{\underset{\sim}{\theta}}$ are the same, then standard error of prediction will be zero, and also the latent trait $\underset{\sim}{\theta}$ always has a larger norm than the estimated $\hat{\underset{\sim}{\theta}}$, $\|\underset{\sim}{\theta}\| \geq \|\hat{\underset{\sim}{\theta}}\|$, since the left hand side of Equation (7) is always

nonnegative. In other words, $\underset{\sim}{\theta}$ has larger standard deviation than $\underset{\sim}{\hat{\theta}}$ has.

Multiple R: The correlation of the estimated $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$ itself indicates to what extent $\underset{\sim}{\hat{\theta}}$ represents the unknown vector $\underset{\sim}{\theta}$. In order to show the relationship between these two vectors, some further properties are needed.

The inner product of $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$ is the sum of squared Fourier coefficients:

(8) $$(\underset{\sim}{\theta},\underset{\sim}{\hat{\theta}}) = \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)^2$$

A simple calculation leads us to the relation that the squared norm of $\underset{\sim}{\hat{\theta}}$ is also the sum of squared Fourier coefficients, given as follows:

(9) $$\| \underset{\sim}{\hat{\theta}} \|^2 = \left(\sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)\underset{\sim}{e}_i, \ \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)\underset{\sim}{e}_i \right) = \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)^2$$

since $(\underset{\sim}{e}_i,\underset{\sim}{e}_j) = 0$ for $i \neq j$, $(\underset{\sim}{e}_i,\underset{\sim}{e}_j) = 1$ for $i = j$. Therefore Equation (10) below, the relation that the norm of $\underset{\sim}{\theta}$ equals the inner product of $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$, is true.

(10) $$(\underset{\sim}{\theta},\underset{\sim}{\hat{\theta}}) = \| \underset{\sim}{\hat{\theta}} \|^2$$

Let us denote the covariance of $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$ by $\mathrm{cov}(\underset{\sim}{\theta},\underset{\sim}{\hat{\theta}})$, which is the inner product of $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$ divided by N-1, and the standard deviation of $\underset{\sim}{\theta}$ by $\sigma(\underset{\sim}{\hat{\theta}})$.

(11) $$\mathrm{cov}(\underset{\sim}{\theta},\underset{\sim}{\hat{\theta}}) = \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)^2/N-1)$$

(12) $$\sigma^2(\underset{\sim}{\hat{\theta}}) = \| \underset{\sim}{\hat{\theta}} \|^2/(N-1)$$

Hence,

(13) $\quad \operatorname{cov}(\underset{\sim}{\theta},\hat{\underset{\sim}{\theta}}) = \sigma^2(\hat{\underset{\sim}{\theta}})$

The squared correlation between $\underset{\sim}{\theta}$ and $\hat{\underset{\sim}{\theta}}$ is given by

(14) $$R^2 = \operatorname{corr}^2(\theta,\hat{\theta}) = \frac{1}{\|\underset{\sim}{\theta}\|^2}\sum_{i=1}^{n}(\underset{\sim}{\theta},\underset{\sim}{e}_i)^2$$

Meanwhile, the squared correlation between $\underset{\sim}{\theta}$ and $\underset{\sim}{e}_i$ is given by

$$R_i^2 = \operatorname{corr}^2(\underset{\sim}{\theta},\underset{\sim}{e}_i) = \frac{1}{\|\underset{\sim}{\theta}\|^2}(\underset{\sim}{\theta},\underset{\sim}{e}_i)^2$$

Therefore, the squared multiple R of the multiple regression $\hat{\underset{\sim}{\theta}}$ equals to the sum of the squared correlation between $\underset{\sim}{\theta}$ and $\underset{\sim}{e}_i$, thus Equation (15) holds:

(15) $$R^2 = R_1^2 + \ldots + R_n^2$$

Let us change the notation of the component R_i in Equation (15), to $R_{\theta e_i}$ so that the distinction from $R_{\hat{\theta}e_i}$, the correlation between $\hat{\underset{\sim}{\theta}}$ and $\underset{\sim}{e}_i$, will be more noticeable. It can be shown that $R_{\hat{\theta}e_i}$ is given by the following formula,

$$R_{\hat{\theta}e_j}^2 = \frac{(\underset{\sim}{\theta},\underset{\sim}{e}_j)^2}{\sum(\underset{\sim}{e}_i,\underset{\sim}{\theta})^2} = \frac{(\underset{\sim}{\theta},\underset{\sim}{e}_j)^2}{\|\hat{\underset{\sim}{\theta}}\|^2}$$

The numerators of $R_{\theta e_j}$ and $R_{\hat{\theta}e_i}$ are same but the denominators are the norms of θ and $\hat{\theta}$ respectively. It should be noted that the correlation of $\underset{\sim}{\theta}$ and $\underset{\sim}{e}_i$, $R_{\theta e_j}$ is smaller or equal to the correlation of $\hat{\underset{\sim}{\theta}}$ and $\underset{\sim}{e}_j$, $R_{\hat{\theta}e_j}$ because $\|\hat{\theta}\| \leq \|\theta\|$. That is, $R_{\hat{\theta}e_j} \geq R_{\theta e_j}$, the correlation of estimated $\underset{\sim}{\theta}$ with each $\underset{\sim}{e}_i$ is always larger than or equal to the correlation of $\underset{\sim}{\theta}$ itself and each $\underset{\sim}{e}_i$. $R_{\hat{\theta}e_j}^2$ will be equal to $R_{\theta e_j}^2$ if and only if $\|\hat{\underset{\sim}{\theta}}\| = \|\underset{\sim}{\theta}\|$.

Complete Hilbert space: If $\{\underset{\sim}{e}_i\}$ is an orthonormal set in a Hilbert space, then Hölder's inequality, Equation (16) will be always true.

$$(16) \qquad \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)^2 \leq \| \underset{\sim}{\theta} \|^2$$

The proof of this theorem will be given first by taking the squared norm of $\underset{\sim}{\theta} - \sum (\underset{\sim}{\theta},\underset{\sim}{e}_i)\underset{\sim}{e}_i$, that is $\| \underset{\sim}{\theta} - \hat{\underset{\sim}{\theta}} \|^2$, we obtain

$$\| \underset{\sim}{\theta} - \sum_{i=1}^{n} (\underset{\sim}{\theta},\underset{\sim}{e}_i)\underset{\sim}{e}_i \|^2 = \| \underset{\sim}{\theta} \|^2 - \| \hat{\underset{\sim}{\theta}} \|^2$$

Since the left-hand side of the above equation is always positive, so inequality (16) is always true for $\{\underset{\sim}{e}_i\}$ and $\underset{\sim}{\theta}$, but the result can be generalized to any vector $\underset{\sim}{y}$ and orthonormal set.

It is interesting to note that the norm of an estimated $\hat{\underset{\sim}{\theta}}$ value is always smaller than or equal to the norm of $\underset{\sim}{\theta}$ itself. The degree of accuracy in estimating $\underset{\sim}{\theta}$ can be measured by the difference of the norms. Geometrically, $\| \underset{\sim}{\theta} \|$ means that the endpoint of $\underset{\sim}{\theta}$ is on a circle of radius $\| \underset{\sim}{\theta} \|$ with the origin as its center. Hence the magnitude of $\| \underset{\sim}{\theta} \|^2 - \| \hat{\underset{\sim}{\theta}} \|^2$ is the extent which the circle of radius $\| \hat{\underset{\sim}{\theta}} \|$ is close to the circle of radius $\| \underset{\sim}{\theta} \|$. Since the standard error of prediction is given by $\sqrt{\| \underset{\sim}{\theta} \|^2 - \| \hat{\underset{\sim}{\theta}} \|^2}$, it can be restated geometrically that the difference of the areas of the two circles with radii $\| \underset{\sim}{\theta} \|$ and $\| \hat{\underset{\sim}{\theta}} \|$ is proportional to the squared standard error of prediction.

We know that the squared multiple R of $\underset{\sim}{\theta}$ and $\underset{\sim}{e}_i$, $R^2_{\theta e_i}$ is generally smaller than the squared multiple R of $\hat{\underset{\sim}{\theta}}$ with $\underset{\sim}{e}_i$, $R^2_{\hat{\theta} e_i}$. This relation implies that the correlation of $\hat{\underset{\sim}{\theta}}$ with each $\underset{\sim}{e}_i$ tends to provide a bit inflated

R value compared with the correlation of the latent trait variable $\underset{\sim}{\theta}$ itself and with $\underset{\sim}{e}_i$. The squared correlation of $\underset{\sim}{\theta}$ and $\underset{\sim}{\hat{\theta}}$ is expressed as the sum of the squared correlation of $\underset{\sim}{\theta}$ with each element of $\{\underset{\sim}{e}_i\}$ according to Equation (15). If we replace each term of Equation (15) by the estimated squared correlation, $R^2_{\hat{\theta}e_i}$, then the newly obtained estimated $R^2_{\theta\hat{\theta}}$ will be larger than the original squared multiple R. This fact warns us that the estimated R^2 is inflated if we use the estimated $\underset{\sim}{\hat{\theta}}$ instead of $\underset{\sim}{\theta}$ itself in the practice.

The concept of the complete latent trait space for n items in the theory of latent trait models implies that the parameters obtained from the models are population free and should not be subject to either sampling errors or errors of measurement in the Classical Test Theory sense. Lord and Novick (1969) showed derivations of the equivalent properties of the assumption of local independence in Chapter 16 (pp. 358-362) in their book. I think the argument in this chapter is somewhat misleading because they introduced the concept of the complete latent trait space with the context of multidimensional complete latent trait space and, at the same time, discussed about the assumption of local independence there. The local independence is valid only in the unidimensional complete latent trait space so far as the discussion in the book is concerned, and not in multidimensional space. Moreover, the subsequent discussions and theories in Chapters 17 through 19 are really only for a unidimensional space.

If we accept the existence of such a unidimensional complete latent trait space for n items and the assumption of local independence, then we are supposed to be able to obtain the sample-free and population-free parameters

a's and b's theoretically. One might wonder whether or not this nice feature can be extended to our model.

Let us restate the concept of a unidimensional complete latent trait space in terms of vector analytical context. If the given n items are enough to measure the latent trait θ that will affect the performance on these items in some population, then the vector $\underset{\sim}{\theta}$ lies in the space spanned by n items. Therefore $\underset{\sim}{\theta}$ can be expressed as a linear combination of these n vectors, $\{\underset{\sim}{x}_i\}$. An orthonormal set $\{\underset{\sim}{e}_i\}$ in Hilbert space is said to be complete if it is maximal, in other words if it is impossible to adjoin a new vector to the set $\{\underset{\sim}{e}_i\}$ in such a way that $\{\underset{\sim}{e}_i, \underset{\sim}{e}_{n+1}\}$ is an orthonormal set which contain $\{\underset{\sim}{e}_i\}$ as a subset. Then such a set $\{\underset{\sim}{e}_i\}$ is said to be complete and if x is an arbitrary vector in the space, then

(17) $$\underset{\sim}{x} = \sum_{i=1}^{n} (\underset{\sim}{x}, \underset{\sim}{e}_i)\underset{\sim}{e}_i$$

and the squared norm of $\underset{\sim}{x}$ is

(18) $$\| \underset{\sim}{x} \|^2 = \sum_{i=1}^{n} (\underset{\sim}{x}, \underset{\sim}{e}_i)^2 .$$

Since $\underset{\sim}{x}$ is an arbitrary vector in the space, and θ lies in the space spanned by $\{\underset{\sim}{x}_i\}$, the $\underset{\sim}{x}$ in Equation (17) can be replaced by $\underset{\sim}{\theta}$. So, Equations (17) and (18) will be rewritten as follows:

(19) $$\underset{\sim}{\theta} = \sum_{i=1}^{n} (\underset{\sim}{\theta}, \underset{\sim}{e}_i)\underset{\sim}{e}_i$$

and the squared norm of $\underset{\sim}{\theta}$ is given by (20),

(20) $$\| \underset{\sim}{\theta} \|^2 = \sum_{i=1}^{n} (\underset{\sim}{\theta}, \underset{\sim}{e}_i)^2$$

Note that Equation (19) is an exact expression of $\underset{\sim}{\theta}$ itself and not the estimated $\hat{\underset{\sim}{\theta}}$. From Equations (20) and (7), the standard error of prediction will be zero in this case. As long as a unidimensional complete latent space for n items exists, then $\underset{\sim}{\theta}$ should be obtained by Equation (19).

<u>Calculation Formula</u>: The estimation of $\underset{\sim}{\theta}$ was given in Equation (6), but note that the Fourier coefficients $(\underset{\sim}{\theta},\underset{\sim}{e}_i)$, i = 1,. . .,n include unknown variable $\underset{\sim}{\theta}$. Therefore they must be approximated by measurable variable. The goal can be achieved first by rewriting $\underset{\sim}{e}_i$ as a linear combination of $\underset{\sim}{x}_i$, then we obtain Equation (21).

$$(21) \qquad e_i = \sum_{k=1}^{i} \alpha_{ik} x_k$$

It should be noted that the coefficients α_{ik} are determined recursively and given as follows,

$$(22) \qquad \alpha_{ik} = -[\sum_{j=k}^{i-1} (\underset{\sim}{x}_i,\underset{\sim}{e}_j)\alpha_{ij}]/K_i$$

Substituting $\underset{\sim}{e}_i$ in Equation (6) by Equation (21), $\underset{\sim}{\theta}$ is expressed as a linear combination of $\underset{\sim}{x}_k$.

$$(23) \qquad \hat{\underset{\sim}{\theta}} = \sum_{i=1}^{n} (\sum_{k=1}^{i} (\underset{\sim}{\theta},\underset{\sim}{e}_i)\alpha_{ik}\underset{\sim}{x}_k)$$

$$= \sum_{k=1}^{n} (\sum_{j=k}^{n}(\underset{\sim}{\theta},\underset{\sim}{e}_j)\alpha_{jk})\underset{\sim}{x}_k$$

where the last member in the above equations is obtained by taking the summation of terms column-wise, while the terms in the second member are summed row-wise.

Next, let us show that the Fourier coefficient $(\underset{\sim}{\theta},\underset{\sim}{e}_j)$ can be approximated by a function of the item discriminating index a_j which were calibrated earlier in a large sample by the MXL estimation method. Without loss of generality, we can assume that the latent trait θ follows the normal distribution of the mean 0 and the standard deviation of unity. Substituting $\underset{\sim}{e}_i$ in $(\underset{\sim}{\theta},\underset{\sim}{e}_i)$ by Equation (21), and taking the summation over k and the coefficients α_{ik} out of the parentheses of the inner product, the middle term of Equation (24) below is obtained. Since $(\underset{\sim}{\theta},\underset{\sim}{x}_k)$ is written by the product of the correlation between $\underset{\sim}{x}_k$ and $\underset{\sim}{\theta}$, $\rho_{\theta k}$, and the standard deviation of $\underset{\sim}{x}_k$, $\sigma(\underset{\sim}{x}_k)$ with the multiplier N-1, thus we obtain the right most term of Equation (24) by substituting the relation for $(\underset{\sim}{\theta},\underset{\sim}{x}_k)$.

$$(24) \qquad (\underset{\sim}{\theta},\underset{\sim}{e}_i) = \sum_{k=1}^{i} \alpha_{ik}(\underset{\sim}{\theta},\underset{\sim}{x}_k) = \sum_{k=1}^{i} \alpha_{ik}\rho_{\theta k}\sigma(\underset{\sim}{x}_k)(N-1).$$

Replacing the Fourier coefficients in Equation (23) by the right most member of Equation (24), the final form for actual calculation is derived as follows:

$$(25) \qquad \hat{\underset{\sim}{\theta}} = (N-1) \sum_{k=1}^{n} [\sum_{j=k}^{n} (\sum_{i=1}^{j} \rho_{\theta x_i}\alpha(\underset{\sim}{x}_i)\underset{\sim}{x}_i\alpha_{ji}\alpha_{jk}$$

Since the correlation of $\underset{\sim}{\theta}$ and $\underset{\sim}{x}_i$ is approximated by (Lord and Novick, 1968)

$$\rho(\underset{\sim}{\theta},\underset{\sim}{x}_i) = \frac{a_i}{\sqrt{1 + a_i^2}}$$

the coefficients of $\underset{\sim}{x}_k$ in Equation (25) are expressed by

$$\beta_k = (N-1) \sum_{j=k}^{n} \sum_{i=1}^{j} \frac{a_i}{\sqrt{1+a_i^2}} \sigma(\underset{\sim}{x}_i)\alpha_{ji}\alpha_{jk}$$

Hence, the βs for earlier terms in the multiple regression equation remain unaltered when subsequent terms are added in a stepwise manner.

Note that x_js are deviation score vectors, hence Equation (25) is written simply by the linear combination of $\underset{\sim}{x}_k$ with coefficients β_k, which is an ordinary multiple regression equation with standardized coefficients.

$$(26) \qquad \hat{\underset{\sim}{\theta}} = \sum_{i=1}^{n} \beta_k \underset{\sim}{x}_k = R(\underset{\sim}{\theta} \mid \underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n)$$

If raw scores are given first, then the mean of each item j will be approximated by Equation (27),

$$(27) \qquad \mu(x) = (1/\sqrt{2\pi}) \int_{\gamma_0}^{\infty} \exp(-t^2/2)dt = \Phi(-b_j a_j/\sqrt{1 + a_j^2}$$

Finally, the squared multiple R of $\underset{\sim}{\theta}$ and $\hat{\underset{\sim}{\theta}}$ is given by Equation (28) as follows:

$$(28) \qquad R^2_{\theta\hat{\theta}} = \sum_{i=1}^{n} (\sum_{k=1}^{i} \alpha_{ik} \|\underset{\sim}{x}_k\| \frac{a_k}{\sqrt{1 + a_k^2}})^2$$

APPLICATION TO ADAPTIVE TESTING

The estimation formula of $\underset{\sim}{\theta}$, Equation (26) is, indeed, the linear multiple regression function of the criterion variable $\underset{\sim}{\theta}$ regressed on the predictors $\underset{\sim}{x}_k$s with standardized β weights. The coefficients β_k should be calculated together when the a_ks and b_ks of the latent trait theory models are calculated. Then, it is easy to estimate an individual latent trait value θ_i by using Equation (26) when an adaptive test is administered. The situation is similar to that in a cross-validation study of multiple regression analysis.

In the adaptive testing situation, an item is chosen so as to maximize the information function of a set of items. In this case, subsequent items will be picked up so as to maximize the increment of $R_{\theta\hat{\theta}}$ given in Equation (28). We should recall that stepwise multiple regression analysis usually does not pick up the best set of items that will yield the maximum value of multiple R among all possible subsets. Also, multiple R is a positively biased statistic, therefore the resulting R value is inflated. Moreover, as we stated in the earlier part of this paper, a multiple R becomes larger than it should be when each component of $R_{\theta e_i}$ is replaced by $R_{\hat{\theta} e_i}$. Since it is well known that the problem of overestimated information function exists in adaptive testing, it is interesting to note that the same kind of problem persists in this new method. However, our approach of estimating $\underset{\sim}{\theta}$ by the least-squares method does avoid the problem of endless iterations that the MXL method encounters for certain response patterns; therefore the estimated $\underset{\sim}{\hat{\theta}}$ values will always be obtainable in the proposed method.

The following example shows how the new estimation method works with live data that was collected on the PLATO system in the past few years.

<u>Example</u>: A 48-item matrix algebra test was administered to the class of an intermediate statistics course at the University of Illinois during 1976. The data was gathered and calibrated by the MXL method. Since the test was designed to eliminate guessing as much as possible, as evidenced by its having an α_{21} of .947, the two parameter logistic model was used. Four out of the 48 items were omitted in the calibration procedure but none of 83

examinees was omitted. The values of θ estimated by the two methods with various combinations of items are calculated and the results are summarized in Table 1.

Table 1

Comparison of Maximum Likelihood (MXL) and Least-Squares (LS) Estimation Methods in the Matrix Algebra Pretest Items

Items	Multiple R^*	# of cases in which iterations for θ do not converge: LS	MXL	D^{**}	Correlation between the θs obtained by LS and MXL	Standard Deviation of θs by MXL
35-41	.970	0	24	.060	.974	.939
41-44, 47,48	.764	0	36	.102	.947	.855
12-16	.721	0	20	.153	.968	.889
31-35	.773	0	27	.064	.957	.814
12-21	.891	0	12	.069	.952	.848
18-21, 23-25, 29-31	.930	0	10	.085	.933	.817
22-31	.807	0	10	.146	.919	.904

*Multiple R is also the standard deviation of LS estimates since θ is rescaled so as to have a normal distribution with mean 0 and standard deviation one.

**The summation of the squared difference of estimated θ by the LS method from the MXL method over the number of subjects:

$$D = \sum(LS\text{-}MXL)^2/N$$

The multiple Rs of the linear regression equations of $\tilde{\theta}$ onto 7 different sets of items are given in the second column of Table 1. The

number of cases (i.e., response patterns) for which iterations for $\underset{\sim}{\hat{\theta}}$ did not converge are shown in columns 3 and 4 for the LS and MXL methods, respectively. The estimated $\underset{\sim}{\theta}$ values are obtained for all examinees by the LS method, while 12 to 43 percent of the examinees' response patterns led to nonconvergence by the MXL method.

Table 2 shows the individual $\underset{\sim}{\hat{\theta}}$ values when items 12 through 21 were chosen as the predictors of $\underset{\sim}{\theta}$. If a response pattern did not lead to convergence, then the program written by J. B. Sympson sets the value of $\underset{\sim}{\hat{\theta}}$ to either +5 or -5.

In order to measure the closeness of the $\underset{\sim}{\hat{\theta}}$ values estimated by the two methods, the squared differences are summed and divided by N. These values are shown in column 5 of Table 1. The correlation of the two $\underset{\sim}{\hat{\theta}}$ values were also calculated and are shown in column 6. Figure 1 displays the relation of $\underset{\sim}{\hat{\theta}}$ estimated by the MXL and LS methods.

Since our model assumed $\underset{\sim}{\theta}$ to be normally distributed with zero mean and unit standard deviation, the original item discriminating powers (a's) and difficulties (b's) of the two-parameter logistic model were adjusted so that the $\underset{\sim}{\theta}$ there follows the same distribution. Table 3 shows the list of adjusted a's and b's obtained from the two-parameter logistic model, and β weights, for items 12 through 21.

SUMMARY

A latent trait variable was estimated by the least-squares method, in other words by regressing $\underset{\sim}{\theta}$ onto a set of observed scores on given test

Table 2

Values of θ Estimated by MXL and LS
using Items 12 through 21

#	LS	MXL	#	LS	MXL	#	LS	MXL
1	1.2494	1.4402	29	0.0242	-0.1744	57	-1.2829	-1.5943
2	0.4881	0.5006	30	0.4872	0.9962	58	0.2303	0.1992
3	0.7336	0.5912	31	0.8262	0.6034	59	0.9119	1.1607
4	-1.6075	-5.0000	32	1.2365	1.8852	60	0.0953	0.0601
5	-0.3268	-0.1633	33	0.9992	1.1999	61	0.2291	-0.0370
6	0.8280	0.8827	34	-1.6075	-5.0000	62	0.9119	1.1607
7	-1.0224	-0.8850	35	0.0608	-0.0200	63	-0.2867	-0.4722
8	-1.0224	-0.8850	36	0.2845	0.2772	64	0.4442	0.0059
9	-0.3003	-0.3630	37	1.2229	5.0000	65	0.0737	-0.1128
10	-1.6075	-5.0000	38	0.4062	0.3835	66	-0.6160	-0.5620
11	-0.8853	-1.0096	39	0.8545	0.5872	67	-0.1050	-0.4666
12	0.2845	0.2772	40	1.0175	0.9856	68	1.0440	0.6641
13	-0.7244	-0.4572	41	1.2229	5.0000	69	1.2365	1.8852
14	-1.6075	-5.0000	42	-0.1858	-0.5673	70	-1.0813	-0.6937
15	0.0378	-0.2827	43	0.4216	0.4777	71	-0.7567	-0.4943
16	-0.9215	-1.0077	44	0.6219	0.3874	72	0.8144	1.0902
17	1.2365	1.8852	45	-0.9821	-0.9083	73	-0.3282	-0.5923
18	-0.4654	-0.7830	46	-0.5066	-0.4120	74	-1.3838	-1.3554
19	-1.6075	-5.0000	47	0.4041	0.3360	75	-0.3191	-0.5094
20	-0.0023	0.0281	48	0.0378	-0.2827	76	-0.7997	-0.5020
21	-0.4860	-0.1790	49	-0.9480	-0.7564	77	1.2229	5.0000
22	0.7071	0.8879	50	0.4062	0.3835	78	-0.5737	-0.6684
23	-0.2600	-0.3813	51	0.4481	0.2478	79	1.2365	1.8852
24	0.9119	1.1607	52	-0.0954	-0.2269	80	0.1150	-0.1950
25	-0.3559	-0.6202	53	1.2229	5.0000	81	-1.2147	-1.1420
26	1.2229	5.0000	54	-1.2829	-1.5943	82	0.6388	0.5829
27	-1.6075	-5.0000	55	1.0175	0.9856	83	1.2229	5.0000
28	-1.2726	-0.9921	56	-0.9119	-0.7588			

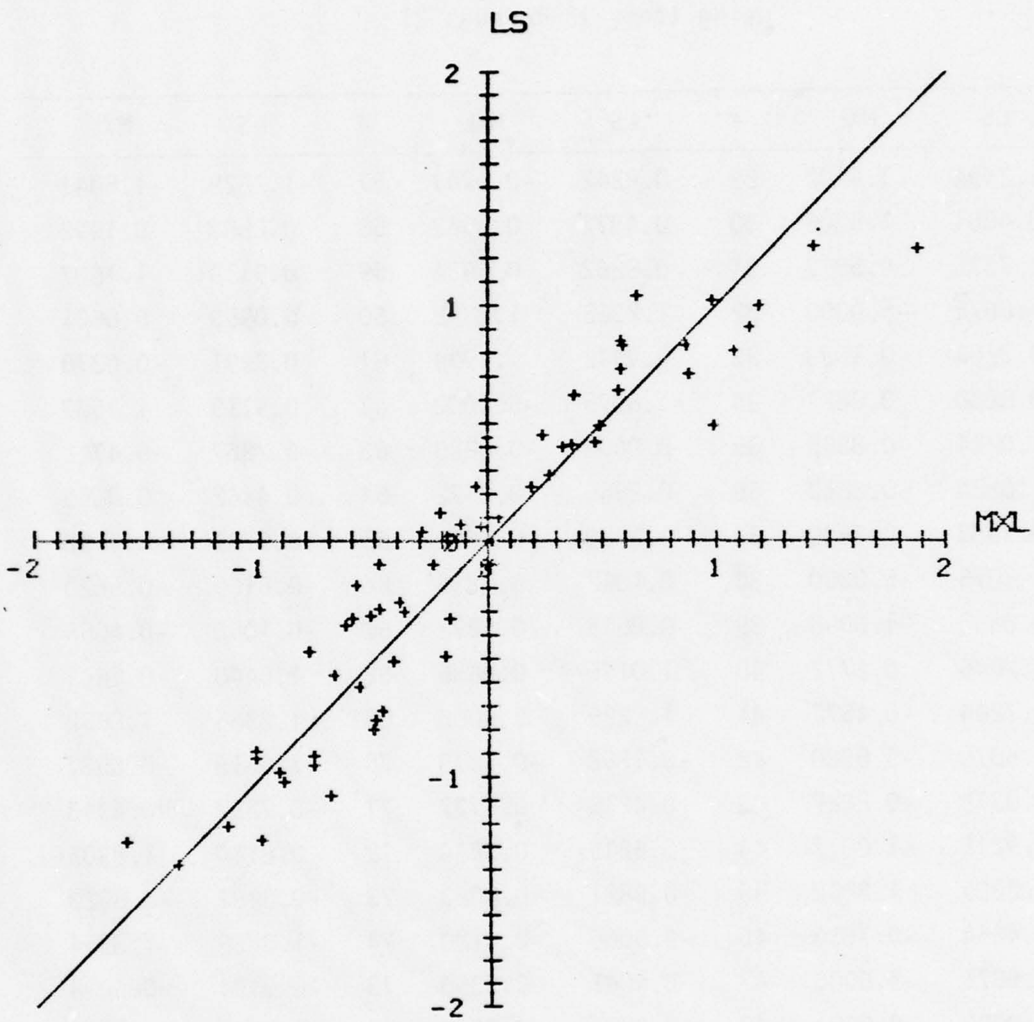

Figure 1

Comparison of θs Estimated by the MXL and LS Methods

Table 3

β Weights of the Least-Squares Estimation Method

Items	a^*	b^*	β
12	.269	1.049	-.014
13	.497	-.025	-.027
14	.469	-.213	.325
15	.699	-.483	.224
16	.733	-.864	.361
17	.815	.485	.409
18	.929	-.619	.736
19	.674	.485	.219
20	.712	.014	.406
21	.608	.137	.191

*a's and b's from the two-parameter logistic model were adjusted so that θ is normally distributed with zero mean and unit standard deviation.

items. It was demonstrated that the standardized β_i coefficient for an item entered at any stage in the linear regression equation for $\underset{\sim}{\theta}$ can be calculated from a sequence of recursively defined intervening variable α_{ik}; the earlier βs remain unchanged in the process. The β_i are also functions of item discriminating power a_i in the two-parameter latent trait theory model that have been calibrated previously. The constant term of the linear multiple regression equation is a function of a's and b's.

If β weights are calculated at the same time as a's and b's in the two-parameter latent trait theory model are calibrated, then the least-squares estimation of $\underset{\sim}{\theta}$ will be easily and always obtained for any different set of items which is given to a student, as in an adaptive testing situation.

The advantage of using the least-squares method as against the maximum likelihood method is that values of $\underset{\sim}{\hat{\theta}}$ are always obtainable even for a small number of examinees and a small number of items, while the latter method cannot provide convergent $\underset{\sim}{\hat{\theta}}$ values too often in such cases.

The accuracies of the estimations were compared and it was concluded that the least-squares method is just as good as the maximum likelihood method. But it should be noted that since the comparison was made between estimated $\underset{\sim}{\hat{\theta}}$s from the LS and MXL methods, there is no way of knowing the exact degree of accuracy of the new method in estimating the true value of a latent trait variable θ. A Monte Carlo study would provide an answer to this question.

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